

2.3 THE DYNAMIC THEORY OF TIDES

When Newton formulated the equilibrium theory of tides in the seventeenth century, he was conscious that it was only a static treatment of the problem and thus only a rough approximation. He was well aware of discrepancies between the predicted equilibrium tides and the observed tides, but did not pursue the matter any further. The equilibrium theory is of limited practical value, even though certain of its predictions are correct, notably that spring and neap tides will occur at new and full Moon (Figure 2.12), that the range of spring tides will typically be two or three times that of neap tides (cf. Figure 2.1), and that tidal inequality is related to declination (Figure 2.9).

There are a number of reasons why actual tides do not behave as equilibrium tides.

- 1 As discussed earlier, the wavelength of tidal waves is long relative to depth in the oceans, so *they travel as shallow-water waves* (Section 2.1) and as we have seen (Question 2.2) their speed is governed by $c = \sqrt{gd}$ (Equation 1.4). The speed of any wave longer than a few km is therefore limited to about 230 m s^{-1} in the open ocean, less in shallower seas. This is much slower than the linear velocity of the surface of the rotating Earth with respect to the Moon: 448 m s^{-1} at the Equator (Question 2.2(a)). (In fact, this linear velocity decreases with distance from the Equator: to $c. 230 \text{ m s}^{-1}$ at about latitudes 60° N and 60° S , to 78 m s^{-1} at 80° latitude, and zero at the poles themselves.)
- 2 In any case, the Earth rotates on its axis far too rapidly for either the inertia of the water masses or the frictional forces at the sea-bed to be overcome fast enough for an equilibrium tide to occur. A time-lag in the oceans' response to the tractive forces is thus inevitable, i.e. there is a tidal *lag*, such that high tide commonly arrives some hours after the passage of the Moon overhead. Because the linear velocity of the surface of the Earth with respect to the Moon decreases polewards (cf. (1) above), the tidal lag is greatest at low latitudes (*c.* 6 hours), decreasing to zero at about latitude 65° – but the precise lag is always constant for a particular location. In addition, at most localities, spring tides occur a day or two after both full and new Moon (cf. Figure 2.12), and the time difference (in days) between the meridian (overhead) passage of full or new Moon and the occurrence of the highest spring high tide is sometimes called the *age of the tide*.
- 3 The presence of land masses prevents the tidal bulges from directly circumnavigating the globe, and the shape of the ocean basins constrains the direction of tidal flows. In fact, the only region of the oceans where a westward-moving tidal bulge could travel unimpeded around the world is the Southern Ocean surrounding Antarctica.
- 4 Except at the Equator, all lateral (horizontal) water movements (including tidal currents) are subject to the **Coriolis force**, which deflects winds and currents *cum sole* (literally 'with the Sun'), i.e. to the right, or clockwise, in the Northern Hemisphere, and to the left, or anticlockwise, in the Southern Hemisphere.

The **dynamic theory of tides** was developed during the eighteenth century by scientists and mathematicians such as Bernoulli, Euler and Laplace. They attempted to understand tides by considering ways in which the depths and configurations of the ocean basins, the Coriolis force, inertia, and frictional forces might influence the behaviour of fluids subjected to rhythmic forces resulting from the orbital relationships of Earth, Moon and Sun.

As a consequence of the many and varied factors involved, the dynamic theory of tides is intricate, and solutions of the equations are complex. Nevertheless, the dynamic theory has been steadily refined, and computed theoretical tides are very close approximations to the observed tides.

The combined constraint of ocean basin geometry and the influence of the Coriolis force (items 2 and 4 on p. 66) results in the development of **amphidromic systems**, in each of which the crest of the tidal wave at high water circulates around an **amphidromic point** once during each tidal period (Figures 2.14 and 2.15 overleaf). The tidal range is zero at each amphidromic point, and increases outwards away from it.

In each amphidromic system, **co-tidal lines** can be defined, which link all the points where the tide is at the same stage (or phase) of its cycle. The successive co-tidal lines radiating outwards from the amphidromic point thus indicate the passage of the tidal wave crest around it.

Cutting across co-tidal lines, approximately at right angles to them, are **co-range lines**, which join places having the same tidal range. Co-range lines form more-or-less concentric circles about the amphidromic point, representing larger and larger tidal ranges the further away they are from it. Figure 2.14 shows the amphidromic systems for the North Sea, and Figure 2.15 shows the computed world-wide amphidromic systems for the dominant tidal component resulting from the diurnal influence of the Moon (see also Section 2.3.1).

QUESTION 2.6

(a) Assume that a high tide coincides with the co-tidal lines marked zero (i.e. '0') on Figure 2.14. At what stage of the tidal cycle is:

- 1 The Wash?
- 2 The Firth of Forth?

(b) Which of (1) and (2) has the greater tidal range?

Inspection of Figures 2.14 and 2.15 shows that, with a few exceptions, the tidal waves of amphidromic systems tend to rotate anticlockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. At first sight, this pattern of rotation appears to conflict with the principle that the Coriolis force deflects moving fluid masses *cum sole*, but we need to bear in mind that the direction of motion of tidal waves is not synonymous with the movement of individual parcels of water.

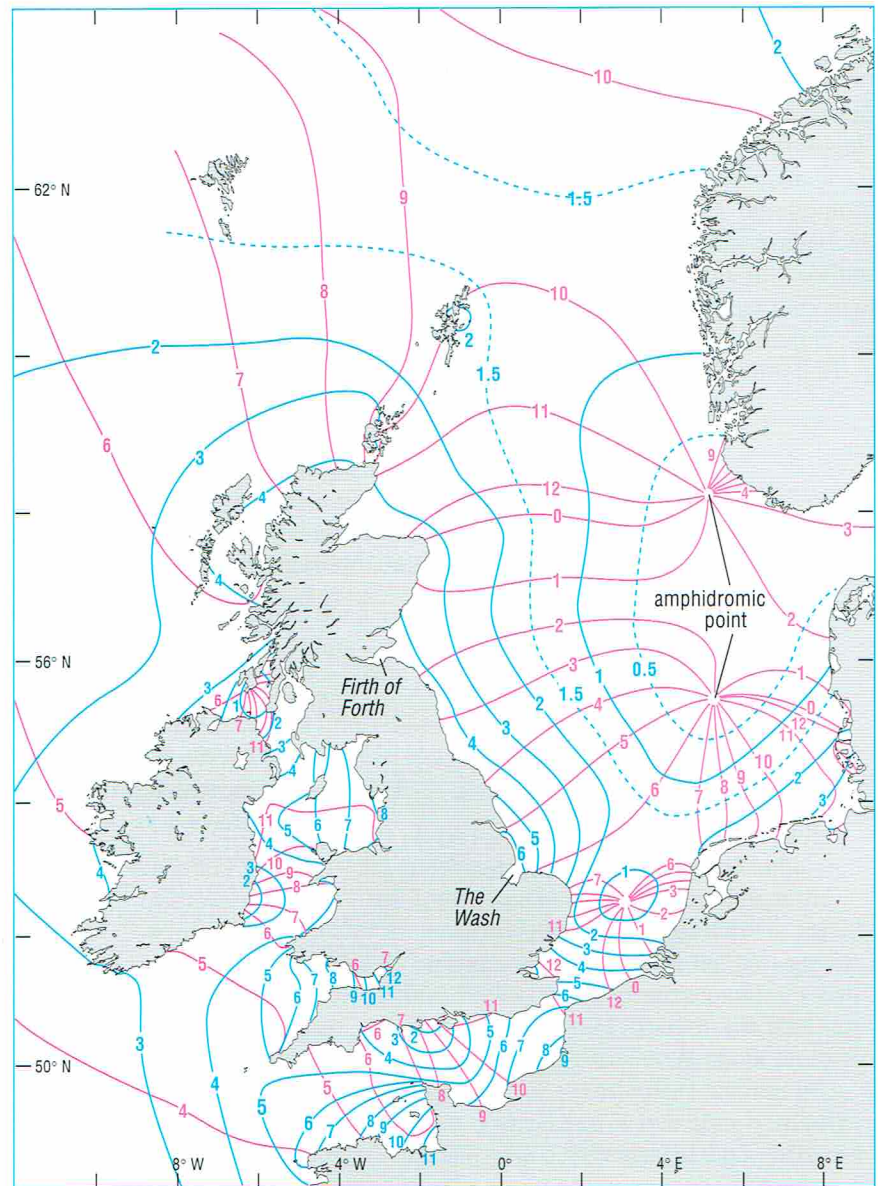


Figure 2.14 Amphidromic systems around the British Isles. The figures on the co-tidal lines (red) indicate the time of high water (in hours) after the Moon has passed the Greenwich meridian. Blue lines are co-range lines, with tidal range in metres.

Consider the enclosed basin shown in Figure 2.16 (on p. 70). The 'bent' arrows in Figure 2.16(a) show how water moving in response to the flooding tide, i.e. in the tidal currents, is deflected to the right by the Coriolis force (the basin is in the Northern Hemisphere), and the water is piled up on the eastern side. Conversely, when the tide ebbs, the water becomes piled up on the western side (Figure 2.16(b)). Hence, because the tidal wave is constrained by land masses, an *anticlockwise* amphidromic system is set up (Figure 2.16(c) and (d)).

It is also very important to remember that tidal waves behave as shallow-water waves, so their orbital motions are flattened like those in Figure 1.8(d). Tidal currents are the horizontal water movements that accompany the rise and fall of the tides as the tidal wave *form* rotates about the amphidromic point, and of course tidal currents change direction during the tidal cycle (see Section 2.4.1).

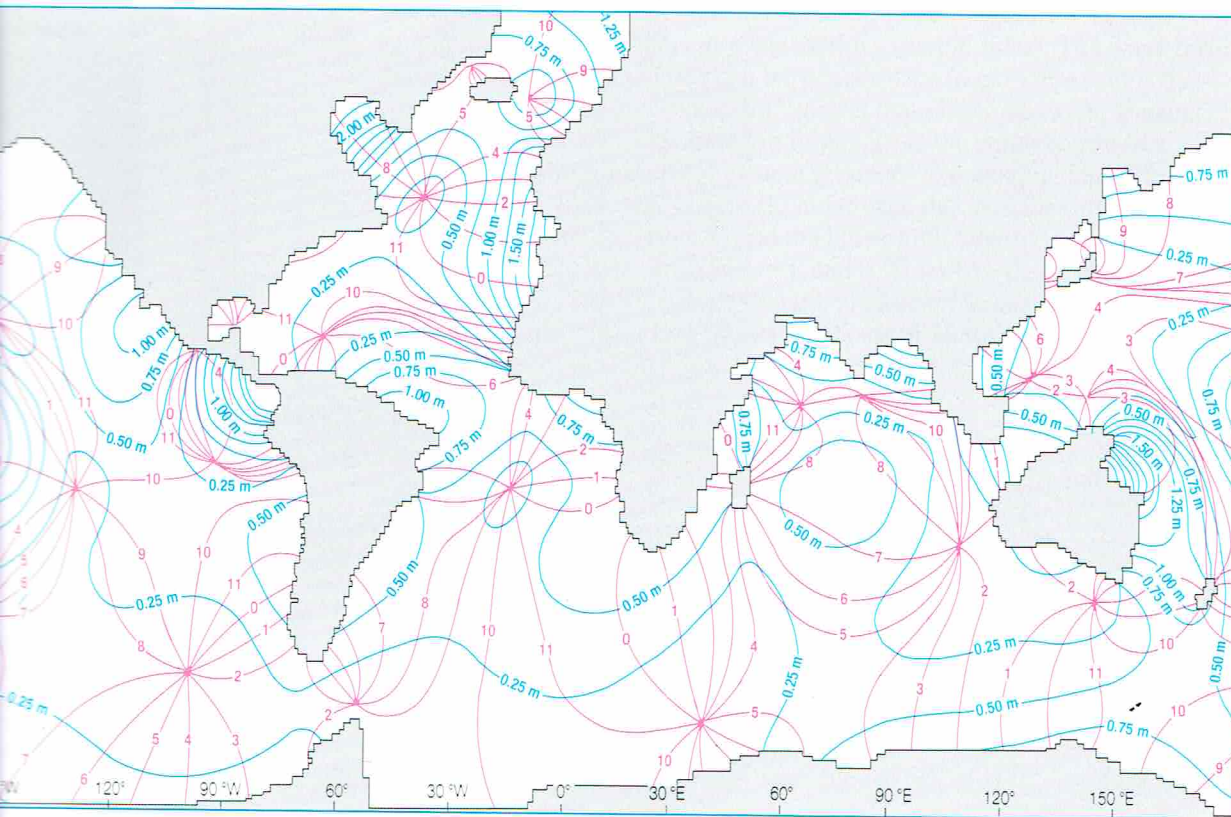


Figure 2.15 Computer-generated diagram of world-wide amphidromic systems for the dominant semi-diurnal lunar tidal component M_2 (see Table 2.1 on p. 71). Blue lines are co-range lines and red lines are co-tidal lines.

The main exceptions to the general pattern of rotation of tidal waves round amphidromic points shown on Figure 2.15 are amphidromic systems less obviously constrained by land masses, e.g. in the South Atlantic (centred on 20° S, 15° W), mid-Pacific (centred on 20° S, 130° W), and North Pacific (centred on 25° N, 155° W); or in certain cases where the amphidromic system rotates about an island, e.g. Madagascar.

QUESTION 2.7

- Locate on Figure 2.15 the amphidromic systems identified above, and state how they are exceptions to the general pattern.
- Locate the amphidromic system centred near 65° E, 5° N in the north-west Indian Ocean. In what way is this also anomalous?

Tidal waves in amphidromic systems are a type of **Kelvin wave**, in which the amplitude is greatest near coasts (Figure 2.16). Kelvin waves occur where the deflection caused by the Coriolis force is either constrained (as at coasts) or is zero (as at the Equator).